In a nutshell: Interpolating polynomials

Given $n + 1$ points $(x_0, y_0), \ldots, (x_n, y_n)$, we can find an interpolating polynomial of degree n that passes through these $n + 1$ points so long as all the x values are distinct.

1. Create the Vandermonde matrix $V = \begin{pmatrix} x_0^n & x_0^{n-1} & \cdots & x_0^2 & x_0 & 1 \\ x_1^n & x_1^{n-1} & \cdots & x_1^2 & x_1 & 1 \\ x_2^n & x_2^{n-1} & \cdots & x_2^2 & x_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ x_{n-1}^n & x_{n-1}^{n-1} & \cdots & x_{n-1}^2 & x_{n-1} & 1 \\ x_n^n & x_n^{n-1} & \cdots & x_n^2 & x_n & 1 \end{pmatrix}$ and the vector $\mathbf{y} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{n-1} \\ y_n \end{pmatrix}$.
2. Solve the system $V\mathbf{a} = \mathbf{y}$.
3. The entries of the solution vector **a** correspond to the coefficient of the term of the corresponding column. Thus, the first entry is the coefficient of x^n, the second of x^{n-1}, and so on, until we get that the second-last entry being the coefficient of the linear term x and the last entry being the constant coefficient.

If these $n + 1$ x-values are equally spaced, we can shift and scale them so that the x-values line up with the points $-n$, $1 - n$, $2 - n$, …, -2, -1, 0, in which case, the Vandermonde matrix becomes:

$$V = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & 1 \\ (-1)^n & (-1)^{n-1} & \cdots & 1 & -1 & 1 \\ (-2)^n & (-2)^{n-1} & \cdots & 4 & -2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ (1-n)^n & (1-n)^{n-1} & \cdots & x_{n-1}^2 & 1-n & 1 \\ (-n)^n & (-n)^{n-1} & \cdots & x_n^2 & -n & 1 \end{pmatrix}$$

Note: Generally, we only find, at most, interpolating polynomials of degree four. Interpolating polynomials can only be used for estimating values between the minimum and maximum x values and should never be used for extrapolation.